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# Free convection in power-law fluids near a three-dimensional stagnation point

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Numerical results are presented for the free-convection boundary-layer equations of the Ostwald de-Waele non-Newtonian power-law type fluids near a three-dimensional (3-D) stagnation point of attachment on an isothermal surface. The existence of dual solutions that are three-dimensional in nature have been verified by means of a numerical procedure. An asymptotic solution for very large Prandtl numbers has also been derived. Solutions are presented for a range of values of the geometric curvature parameter *c*, the power-law index *n*, and the Prandtl number Pr.

**Keywords:** Free convection boundary layer, stagnation point flow, non-Newtonian fluids, rheology

## Introduction

The problem of boundary-layer free convection of a viscous fluid in the vicinity of a three-dimensional (3-D) stagnation point on an isothermal surface has been the object of several studies in the past. For example, such flows have been examined by Poots (1964), Banks (1972), and Ingham et al. (1984). The work of Poots is particularly interesting because he has derived the boundary-layer equations governing the free convection flow at a general 3-D lower stagnation point, and has shown that the two-dimensional (2-D) and axisymmetric flows are just two special cases from a more general point of view. Poots has, in fact, given exact numerical solutions to the 3-D boundary-layer equations, where the Prandtl number was 0.72, for a number of blunt body shapes. Then, Banks has shown that other solutions exist over the whole range of stagnation points and Prandtl numbers. The unsteady free-convection flow of a viscous fluid near 3-D stagnation point on a regular isothermal surface has been analyzed by Ingham et al. Finally, we mention here the papers by Wang (1988) and Ramachandran et al. (1988), who have investigated the effect of free or mixed convection on 2-D and axisymmetric stagnation flows on a vertical plate.

A number of industrially important fluids such as molten plastics, polymers, pulps, foods, and slurries exhibit non-Newtonian fluid behavior. Because of the growing use of these non-Newtonian fluids in various manufacturing and processing industries, considerable efforts have been directed toward understanding their friction and heat transfer characteristics. See the articles by Shenoy and Mashelkar (1982) and Shenoy (1986) for comprehensive reviews.

The object of this paper is to study the problem of free convection of a non-Newtonian fluid obeying the power-law model at a 3-D stagnation point of attachment over an isothermal surface. Similar solutions for the governing boundary-layer equations are obtained. The derived ordinary differential equations are then solved numerically for a wide range of three parameters: the power-law index *n*; the geometric curvature parameter *c*; and the Prandtl number Pr. The overall focus of the analysis is to obtain quantitative information on the effects of these parameters on the flow and heat transfer characteristics. The solutions for plane and axisymmetric flows are two special cases covered herein from a more general point of view. The particular solution for a Newtonian fluid is compared with the published results.

## Governing equations

We consider the steady incompressible laminar boundary layer flow of a non-Newtonian fluid in the vicinity of a 3-D stagnation point on a regular surface under the effect of free convection.

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We take the temperature of the ambient fluid to be $T_\infty$ (a constant), and the body is maintained at a constant temperature $T_w(>T_\infty)$. Moreover, in accordance with previous work reported by Shulman et al. (1975), Shvets and Vishnevskeiy (1987), Gryglaszewski and Saljnikov (1989) and Pop et al. (1993) the following transport properties based on the power-law model are assumed to hold

$$\tau_{ij} = -p\delta_{ij} + K|\tfrac{1}{2}I_2|^{(n-1)/2}e_{ij} \tag{1}$$

$$q = -k|\tfrac{1}{2}I_2|^{s}\nabla T \tag{2}$$

A locally orthogonal set of co-ordinates $(x, y, z)$ is chosen with the stagnation point in question at the origin $O$ (0, 0, 0) and so that $z$ measures distance normal to the surface at $O$. The parametric curves $x =$ constant and $y =$ constant on the surface coincide with the lines of curvature. In this co-ordinate system, the appropriate 3-D boundary-layer equations for the present study can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = g^*\beta(T - T_\infty)ax + \frac{K}{\rho}\frac{\partial}{\partial z}\left(\left|\frac{\partial u}{\partial z}\right|^{(n-1)}\frac{\partial u}{\partial z}\right) \tag{4}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = g^*\beta(T - T_\infty)$$

$$\text{by} \quad + \frac{K}{\rho}\frac{\partial}{\partial z}\left(\left|\frac{\partial u}{\partial z}\right|^{(n-1)}\frac{\partial v}{\partial z}\right) \tag{5}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\frac{\partial}{\partial z}\left(\left|\frac{\partial u}{\partial z}\right|^{(n-1)}\frac{\partial T}{\partial z}\right) \tag{6}$$

As Banks (1972) pointed out, the body is of nodal type at (0, 0, 0) if the parameters $a$ and $b$ are non-negative, and the solutions of Equations 3–6 correspond to stagnation points that are then nodal points of attachment. However, if one of $a$ and $b$ is negative, the body is of saddle type, and it is to be assumed that certain solutions of the equations will correspond to saddle points of attachment. On the other hand, if $a$ and $b$ are negative, the body is again of nodal type, but the flow is one of separation. Without loss of generality, we consider here only the case when the stagnation point is of a nodal point of attachment. It is important to mention that we have assumed that $v \ll u$ and therefore, neglected $(\partial v/\partial z)$ from the apparent viscosity terms in Equations 4–6. Furthermore, we have imposed the condition $s = n - 1$, so that Equations 3–6 reduce to ordinary differential equations (see Pop and Gorla 1990).

The problem is completely posed by adding the boundary conditions

$$u(x, y, 0) = v(x, y, 0) = w(x, y, 0) = 0, \quad T(x, y, 0) = T_w$$

$$u(x, y, \infty) = v(x, y, \infty) = 0, \quad T(x, y, \infty) = T_\infty \tag{7b}$$

We now seek similarity solutions of Equation 3–6 subject to Equation 7 of the form

$$u = U_0(ax)f'(\eta) \tag{8a}$$

$$v = U_0(ay)cg'(\eta) \tag{8b}$$

$$W = -U_0(ax)^{(n-1)/(n+1)}\mathrm{Gr}^{-1/2(n+1)}[\{2n/(n+1)\}f + \{(1-n)/(n+1)\}nf' + cg] \tag{8c}$$

$$h(\eta) = (T - T_\infty)/(T_w - T_\infty) \tag{8d}$$

where

$$\eta = (ax)^{-(n-1)/(n+1)}\mathrm{Gr}^{1/2(n+1)}(az) \tag{9}$$

is the similarity variable. Here $c = b/a$ is a geometric curvature parameter; $U_o = [g^*\beta(T_w - T_\infty)/a]^{1/2}$ is the characteristic velocity; and Gr is the modified Grashof number defined as follows: $\mathrm{Gr} = (\rho/K)^2a^{-(n+2)}[g^*\beta(T_w - T_\infty)]^{2-n}$.

Substituting Equations 8a–d and 9 into Equations 4–6, we obtain

$$(|f''|^{n-1}f'')' + [2n/(n+1)f + cg]f'' - (f')^2 + h = 0 \tag{10}$$

$$(|f''|^{n-1}g'')' + [2n/(n+1)f + cg]g'' - c(g')^2 + ch = 0 \tag{10}$$

$$1/\mathrm{Pr}(|f''|^{n-1}h')' + [2n/(n+1)f + cg]h' = 0 \tag{12}$$

where $\mathrm{Pr} = K/(\rho\alpha)$ is the Prandtl number, and primes denote differentiation with respect to $\eta$. The transformed boundary conditions now become

$$f(0) = f'(0) = g(0) = g'(0) = 0, \quad h(0) = 1 \tag{13a}$$

$$f'(\infty) = g'(\infty) = h(\infty) = 0 \tag{13b}$$

## Notation

| | |
|---|---|
| $a, b$ | curvatures of the body surface measured in the planes $y = 0$ and $x = 0$, respectively |
| $c$ | geometric curvature parameter |
| $e_{ij}$ | strain rate tensor component |
| $f, g$ | dimensionless reduced stream functions |
| $f_1, \hat{f}_1, g_1, h_1$ | functions associated with large Prandtl number |
| $h$ | dimensionless temperature |
| $F, G, H$ | functions associated with dual solutions |
| $g^*$ | acceleration due to gravity |
| Gr | modified Grashof number |
| $I_2$ | second invariant of the strain rate tensor |
| $k$ | modified thermal conductivity |
| $K$ | consistency index |
| $n$ | fluid power-law index |
| Nu | Nusselt number |
| $p$ | pressure |
| Pr | Prandtl number |
| $q$ | heat flux |
| $s$ | heat transfer power-law index |
| $T$ | temperature |
| $u, v, w$ | velocity components along $(x, y, z)$ directions |
| $U_0$ | characteristic velocity |
| $x, y, z$ | local orthogonal coordinates with $x$ and $y$ axes along the body surface and the coordinate $z$ normal to the surface |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | modified thermal diffusivity |
| $\beta$ | volumetric expansion coefficient |
| $\delta_{ij}$ | unit tensor |
| $\eta, \xi, \zeta$ | similarity variables |
| $\rho$ | density |
| $\tau_{ij}$ | stress tensor component |

*Subscripts*

| | |
|---|---|
| $w$ | refers to surface values |
| $\infty$ | refers to values in the ambient fluid |

It is important to note at this point that for $c = 0$, we obtain the 2-D problem by assuming $g = 0$; whereas, for $c = 1$ and assuming $f = g$ results in the axisymmetric problem for this flow configuration.

Once the similarity functions $(f, g, h)$ are known, the Nusselt number can be defined as follows:

$$\mathrm{Nu} = q_w/[ka(T_w - T_\infty)(aU_0)^{n-1}] \tag{14}$$

where $q_w$ is determined from Equations 2. Using here Equations 8 and 9, we can obtain a heat transfer group, which is given by the following:

$$\mathrm{Nu}\mathrm{Gr}^{-n/2(n+1)}(ax)^{-(n-1)/(n+1)} = -|f''(0)|^{n-1}h'(0) \tag{15}$$

## Dual solutions

We show in this section that for $c = 0$, a solution exists of Equations 10–12 subject to Equation 13, the resulting flowfield being of a 3-D nature. Thus, when $c = 0$, and assuming $g = 0$, Equations 10 and 12 become

$$(|f''|^{n-1}f'')' + 2n/(n+1)ff'' - (f')^2 + h = 0 \tag{16}$$

$$1/\mathrm{Pr}(|f''|^{n-1}h')' + 2n/(n+1)fh' = 0 \tag{17}$$

subject to

$$f(0) = f'(0) = 0, \quad h(0) = 1 \tag{18a}$$

$$f'(\infty) = h(\infty) = 0 \tag{18b}$$

Furthermore, following Banks (1972), we show that another solution of Equations 10–13 is possible by first writing $G = cg$ and $(F, H)$ instead of $(f, h)$. Substituting these forms into Equations 10–12, and taking the limit as $c \to 0$, we obtain

$$(|F''|^{n-1}F'')' + [2n/(n+1)F + G]F' - (F')^2 + H = 0 \tag{19}$$

$$(|F''|^{n-1}G'')' + [2n/(n+1)F + G]G'' - (G')^2 = 0 \tag{20}$$

$$\frac{1}{\mathrm{Pr}}(|F''|^{n-1}H')' + [2n/(n+1)F + G]H' = 0 \tag{21}$$

subject to the same boundary conditions as in Equation 13. It should be mentioned here that the velocity in the third dimension (i.e., in the *oy*-direction) is, by virtue of Equation (8b), given by

$$v = U_0(ay)G'(\eta) \tag{22}$$

and so is a non-zero physical component, provided a non-zero function $G'(\eta)$ exists. Now, although $G = 0$ is clearly one solution, the numerical solution of Equation 20 shows that another solution of this equation exists.

## Large Prandtl number solution

When Pr is large, we introduce variables

$$\xi = \mathrm{Pr}^{n/2(n+1)}\eta \tag{23}$$

$$f_1(\xi) = \mathrm{Pr}^{(2n+1)/2(n+1)}f(\eta) \tag{24a}$$

$$g_1(\xi) = \mathrm{Pr}^{(2n+1)/2(n+1)}g(\eta), \quad h_1(\xi) = h(\eta) \tag{24b}$$

From Equations 10–12, we then have the following:

$$(|f_1''|^{n-1}f_1'')' + h_1 = \mathrm{Pr}^{-1}\{(f_1')^2 - [2n/(n+1)f_1 + cg_1]f_1''\} \tag{25}$$

$$(|f_1''|^{n-1}g_1'')' + ch_1 = \mathrm{Pr}^{-1}\{c(g_1')^2 - [2n/(n+1)f_1 + cg_1]g_1''\} \tag{26}$$

$$(|f_1''|^{n-1}h_1')' + [2n/(n+1)f_1 + cg_1]h_1' = 0 \tag{27}$$

where primes now imply differentiation with respect to $\xi$. The boundary conditions are the same as in Equation 13. Letting $\mathrm{Pr} \to \infty$, we obtain the following:

$$(|f_1''|^{n-1}f_1'')' + h_1 = 0 \tag{28}$$

$$(|f_1''|^{n-1}g_1'')' + ch_1 = 0 \tag{29}$$

and Equation 27 remains unchanged. The boundary conditions for these equations are as follows:

$$f_1(0) = f_1'(0) = g_1(0) = g_1'(0) = 0, \quad h_1(0) = 1 \tag{30a}$$

$$f_1''(\infty) = g_1''(\infty) = h_1(\infty) = 0 \tag{30b}$$

We note that the boundary conditions $f_1'(\infty) = 0$ and $g_1'(\infty) = 0$ resulting from Equation 13 cannot be applied for Equations 27–29, and instead, we impose zero shear stress at $\xi = \infty$. The physical reason for this assumption is because as Pr becomes very large, the thermal layer becomes much thinner than the momentum layer, so that the edge of the thermal layer the velocities are still finite.

From Equations 28 and 29 it follows that

$$h_1 = -(|f_1''|^{n-1}f_1'')', \quad g_1 = cf_1 \tag{31}$$

which are then substituted into Equation 27 to obtain the following:

$$[|f_1''|^{n-1}(|f_1''|^{n-1}f_1'')'']' + \{[2n/(n+1) + c^2\}f_1(|f_1''|^{n-1}f_1'')'' = 0 \tag{32}$$

along with the boundary conditions

$$f_1(0) = f_1'(0) = 0, \quad f_1'''(0) = -1 \tag{33a}$$

$$f_1''(\infty) = f_1'''(\infty) = 0 \tag{33b}$$

Now the following new variables

$$\hat{f}_1(\zeta) = [2n/(n+1) + c^2]^{3/(5-n)}f_1(\xi) \tag{34a}$$

and

$$\zeta = [2n/(n+1) + c^2]^{1/(5-n)}\xi \tag{34b}$$

may be substituted into Equation 32 to eliminate the parameter $c$. After some manipulations, we get the following:

$$[|\hat{f}_1''|^{n-1}(|\hat{f}_1''|^{n-1}\hat{f}_1'')'']' + \hat{f}_1(|\hat{f}_1''|^{n-1}\hat{f}_1'')'' = 0 \tag{35}$$

in which primes now signify differentiation with respect to $\zeta$. The boundary conditions of this equation are seen to be as follows:

$$\hat{f}_1(0) = \hat{f}_1'(0) = 0, \quad \hat{f}_1'''(0) = -1 \tag{36a}$$

$$\hat{f}_1''(\infty) = \hat{f}_1'''(\infty) = 0 \tag{36b}$$

We remark to this end that for $n = 1$ (Newtonian fluid), Equations 35 and 36 reduce to the equations already reported by Banks (1972).

## Results and discussion

Because analytical solutions to Equations 10–12, 19–21, and 35 are unlikely to be obtained, we have resorted to obtaining numerical solutions. The numerical scheme used is the fourth-order Runge–Kutta method. A comprehensive set of results have been obtained covering the ranges $0.4 \le n \le 2.0$ and $0 \le c \le 1$. The Prandtl number has been taken to be 10 and 100. A selection of these results is presented here with a view to isolate the effect of each individual parameter.

Figures 1–3 illustrate velocity and temperature profiles ($f'$, $g'$, $h$) for $c = 0.5$ and $n$ ranging from 0.5 to 1.5. The results indicate that the velocity maximum decreases with $n$ for dilatant fluids ($n > 1$); whereas, for pseudoplastic flows ($n < 1$)

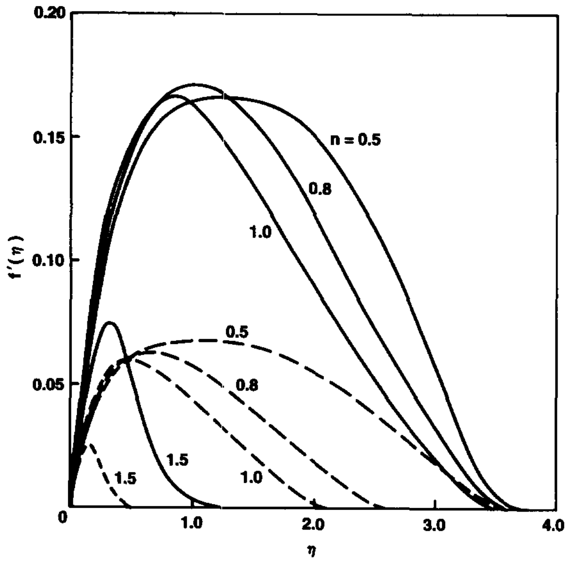


*Figure 1* Velocity profiles $f(\eta)$ for $c = 0.5$ (solid line, Pr = 10; dashed line, Pr = 100)

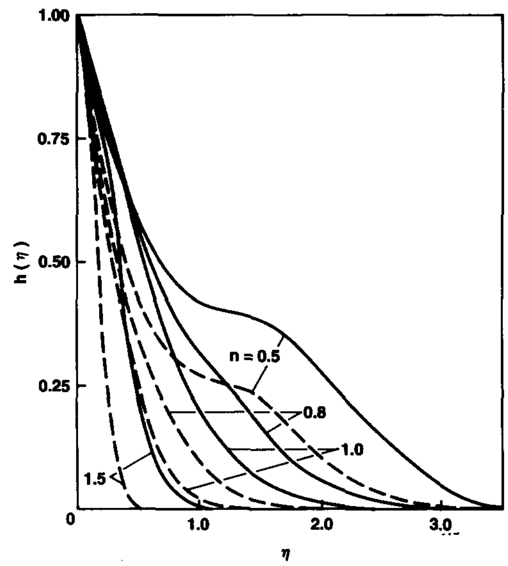


*Figure 3* Temperature profiles $h(\eta)$ for $c = 0.5$ (solid line, Pr = 10; dashed line, Pr = 100)

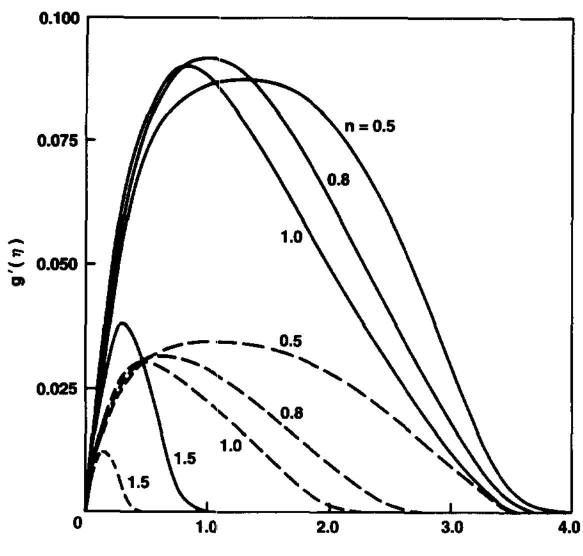


*Figure 2* Velocity profiles $g(\eta)$ for $c = 0.5$ (solid line, Pr = 10; dashed line, Pr = 100)

this quantity is relatively constant. The location of the maximum velocity within the boundary layer is observed to move closer to the wall as the value of $n$ increases. The momentum boundary layer thickness decreases as $n$ increases. The results indicate that the thermal boundary-layer thickness decreases as $n$ increases, and the temperature profiles for the pseudoplastic fluids tend to assume an $s$-shape within the boundary layer, Moreover, it seems that for pseudoplastic fluids, there appears a reverse flow ($f' < 0$ and $g' < 0$) at some distance from the surface.

Table 1 summarizes the wall gradients $f''(0)$, $g''(0)$, and $h'(0)$ from which the friction factor and Nusselt number may be calculated. To examine the accuracy of the present results, we have compared our results with these reported by Banks (1972) and Poots (1964) for the case of a Newtonian fluid ($n = 1$) with Pr = 0.72. This is shown in Figure 4. It is observed that our results are in excellent agreement with those reported in the literature.

Figures 5 and 6 display the results for the variation of the heat transfer group, Nu $Gr^{-n/2(n+1)}\,(ax)^{-(n-1)/(n+1)}$ with the geometric curvature parameter $c$ and the power-law index $n$ for Prandtl numbers of 10 and 100, respectively. It is seen that heat transfer decreases with the increase of $n$ and is greater for $Pr = 100$ than for Pr = 10.

Figures 7–9 display results for the variation of $F'$, $-G'$, and $H$ for a range of values of $n$ with Pr = 100. This represents the case of $c = 0$ where a solution exists in which the resulting flow field is of a 3-D nature, and as such, is quite distinct from the usual 2-D form considered in the literature. The physical meaning of such dual solutions is not well understood, although Schofield and Davey (1967) suggest that these may be interpreted as finite disturbances to the usual solutions and so may be related to the instability of such flows. Experimental verification may shed more light on this problem. For $n = 1$ and Pr = 0.72, our results indicate that $F''(0) = 0.9617$, $G''(0) = -0.2821$, and $H'(0) = -0.2781$. The solution reported by Banks (1972) for these wall functions are 0.96171, −0.28212, and −0.27809 respectively. Table 2 displays our results for $F''(0)$, $G''(0)$, and $H'(0)$ for a range of values of $n$ and Pr. The flow at stagnation points is classified according to the behavior of the skin friction lines on the body surface, and it is clear that the 3-D solution presented here corresponds to a saddle point of attachment.

Figure 10 illustrates the distribution of the velocity profiles $\hat{f}'_1(\zeta)$ within the boundary layer associated with very large values of Pr for a range of values of $n$. It is noticed that there may occur a flow reversal ($\hat{f}'_1 < 0$), and the flow approaches the boundary conditions $f'(\infty) = 0$ and $g'(\infty) = 0$ from below.

Values of $\hat{f}''_1(0)$ and $\hat{f}^{iv}_i(0)$ for some values of $n$ are given in Table 3. It is worth mentioning that the present results for $n = 1$ (Newtonian fluid) are in excellent agreement with those reported by Banks (1972). Thus, our values are 1.0850 and 0.5403; whereas, the literature values are 1.085125 and 0.540235, respectively.

***Table 1*** *Values of* $f''(0)$, $g''(0)$, and $h'(0)$ for a range of values of *c, n,* and Pr

| $n$ | $f''(0)$ | $g''(0)$ | $-h'(0)$ |
|---|---|---|---|
| | | Pr = 0.72, $n$ = 1.0 | |
| $c$ = 0.00 | 0.8549 | 0.0000 | 0.3752 |
| 0.25 | 0.8447 | 0.2565 | 0.3850 |
| 0.50 | 0.8211 | 0.4638 | 0.4075 |
| 0.75 | 0.7926 | 0.6277 | 0.4349 |
| 1.00 | 0.7630 | 0.7629 | 0.4639 |
| | | $c$ = 0.0, Pr = 10.0 | |
| $n$ = 0.5 | 0.6219 | 0.0000 | 1.1773 |
| 0.8 | 0.5526 | 0.0000 | 0.9532 |
| 1.0 | 0.5426 | 0.0000 | 0.8846 |
| 1.5 | 0.5512 | 0.0000 | 0.8034 |
| 2.0 | 0.5745 | 0.0000 | 0.7752 |
| | | $c$ = 0.0, Pr = 100.0 | |
| $n$ = 0.5 | 0.3210 | 0.0000 | 1.8164 |
| 0.8 | 0.3149 | 0.0000 | 1.6826 |
| 1.0 | 0.3273 | 0.0000 | 1.6698 |
| 1.5 | 0.3664 | 0.0000 | 1.7020 |
| 2.0 | 0.4083 | 0.0000 | 1.7992 |
| | | $c$ = 0.25, Pr = 10.0 | |
| $n$ = 0.5 | 0.6035 | 0.1762 | 1.2026 |
| 0.8 | 0.5419 | 0.1489 | 0.9723 |
| 1.0 | 0.5345 | 0.1436 | 0.9015 |
| 1.5 | 0.5455 | 0.1423 | 0.8142 |
| 2.0 | 0.5691 | 0.1465 | 0.7767 |
| | | $c$ = 0.25, Pr = 100.0 | |
| $n$ = 0.5 | 0.3122 | 0.0835 | 1.8447 |
| 0.8 | 0.3096 | 0.0803 | 1.7078 |
| 1.0 | 0.3232 | 0.0829 | 1.6948 |
| 1.5 | 0.3634 | 0.0920 | 1.7493 |
| 2.0 | 0.4028 | 0.1014 | 1.8007 |
| | | $c$ = 0.50, Pr = 10.0 | |
| $n$ = 0.00 | 0.5638 | 0.3096 | 1.2606 |
| 0.25 | 0.5178 | 0.2748 | 1.0186 |
| 0.50 | 0.5147 | 0.2696 | 0.9446 |
| 0.75 | 0.5316 | 0.2735 | 0.8555 |
| 1.00 | 0.5571 | 0.2840 | 0.8077 |
| | | $c$ = 0.50, Pr = 100.0 | |
| $n$ = 0.5 | 0.2904 | 0.1515 | 1.9203 |
| 0.8 | 0.2951 | 0.1510 | 1.7815 |
| 1.0 | 0.3106 | 0.1579 | 0.7691 |
| 1.5 | 0.3529 | 0.1779 | 1.8053 |
| 2.0 | 0.3922 | 0.1970 | 1.8218 |
| | | $c$ = 0.75, Pr = 10.0 | |
| $n$ = 0.5 | 0.5205 | 0.4064 | 1.3290 |
| 0.8 | 0.4887 | 0.3764 | 1.0787 |
| 1.0 | 0.4900 | 0.3753 | 1.0029 |
| 1.5 | 0.5129 | 0.3897 | 0.9047 |
| 2.0 | 0.5431 | 0.4111 | 0.8711 |
| | | $c$ = 0.75, Pr = 100.0 | |
| $n$ = 0.5 | 0.2651 | 0.2026 | 2.0172 |
| 0.8 | 0.2764 | 0.2095 | 1.8832 |
| 1.0 | 0.2935 | 0.2218 | 1.8768 |
| 1.5 | 0.3401 | 0.2561 | 1.9107 |
| 2.0 | 0.3835 | 0.2883 | 2.0054 |
| | | $c$ = 1.0, Pr = 10.0 | |
| $n$ = 0.5 | 0.4800 | 0.4799 | 1.3990 |
| 0.8 | 0.4597 | 0.4597 | 1.3990 |
| 1.0 | 0.4648 | 0.4648 | 1.0677 |
| 1.5 | 0.4928 | 0.4928 | 0.9632 |
| 2.0 | 0.5257 | 0.5256 | 0.9292 |
| | | $c$ = 1.0, Pr = 100.0 | |
| 0.5 | 0.2417 | 0.2417 | 2.1195 |
| 0.8 | 0.2583 | 0.2583 | 1.9950 |
| 1.0 | 0.2775 | 0.2775 | 1.9945 |
| 1.5 | 0.3259 | 0.3259 | 2.0728 |
| 2.0 | 0.3709 | 0.3709 | 2.1598 |

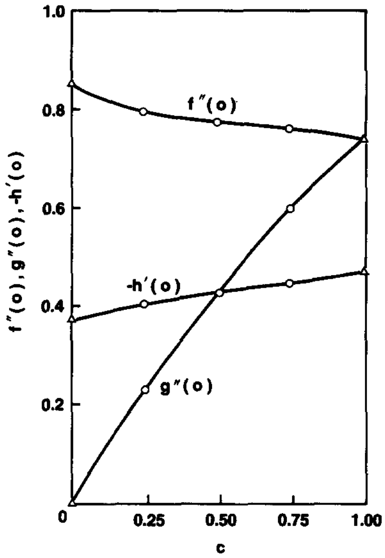


*Figure 4* Values of $f''(0)$, $g''(0)$, and $-h'(0)$ for $n = 1$ (Newtonian fluid) an dPr = 0.72 (solid line, present results; ○ Banks; Δ Poots)

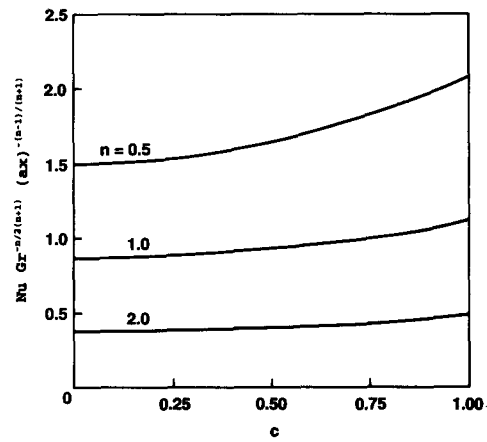


*Figure 5* Variation of heat transfer group with $c$ for Pr = 10

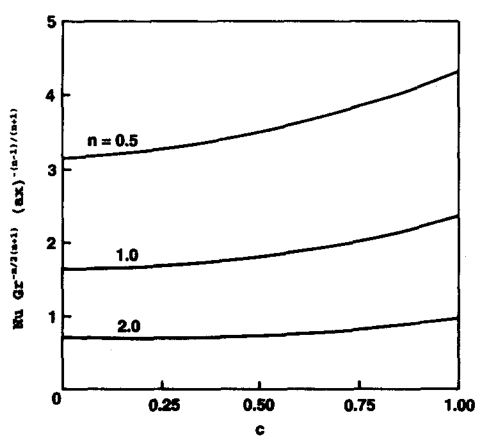


*Figure 6* Variation of heat transfer group with $c$ for Pr = 100

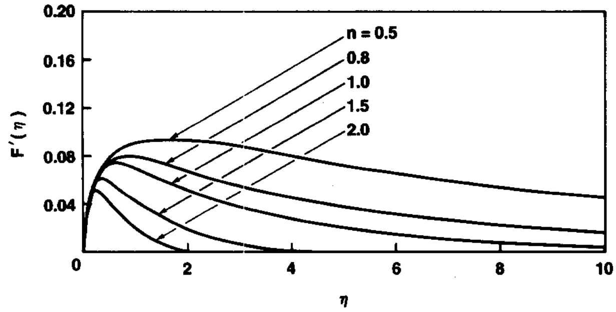


*Figure 7* Dual velocity profiles $F(\eta)$ for $c = 0$ and Pr = 100

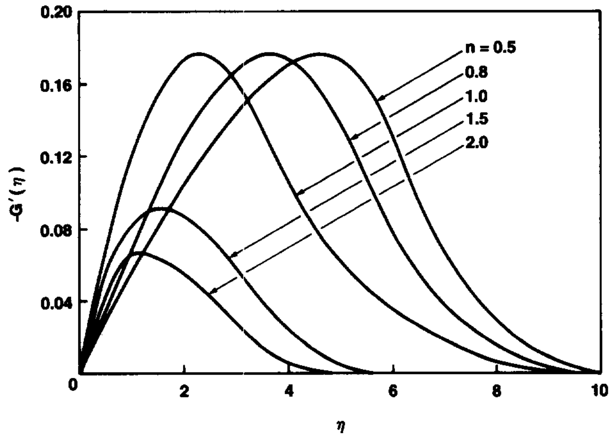


*Figure 8* Dual velocity profiles $-G'(\eta)$ for $c = 0$ and Pr = 100

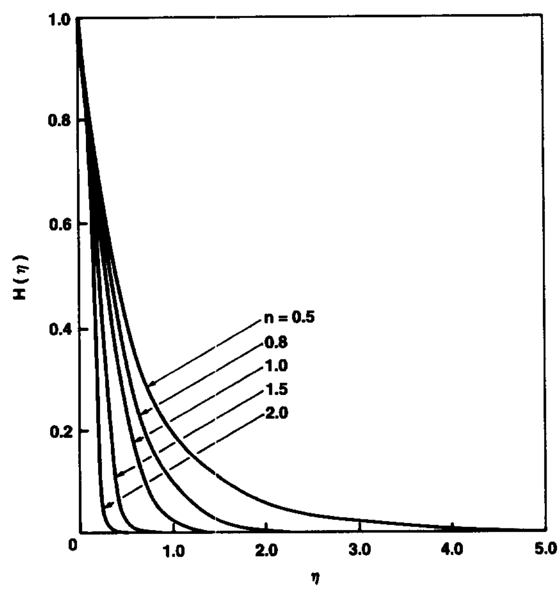


*Figure 9* Dual temperature profiles $H(\eta)$ for $c = 0$ and Pr-100

## Concluding remarks

In this paper, a boundary-layer analysis has been provided for the problem of free convection of a non-Newtonian fluid obeying the Ostwald de-Waele type power-law model in the vicinity of a 3-D stagnation point under isothermal boundary conditions. The existence of a dual solution; namely, a 3-D solution at a 2-D stagnation point has been verified by means of a numerical solution. Finally, we have studied the boundary-layer behavior at a 3-D stagnation point for infinitely large Prandtl numbers. The solutions are obtained for a range of values of the geometric curvature parameter, $c$, the power-law index $n$ and the Prandtl number Pr.

**Table 2** $F''(0)$, $-G''(0)$, and $-H'(0)$ for Pr = 0.72, 10, and 100 for $c = 0$ case

| Pr | $n$ | $F''(0)$ | $-G''(0)$ | $-H'(0)$ |
|---|---|---|---|---|
| 0.72 | 1.0 | 0.9617 | 0.2821 | 0.2781 |
| 10 | 0.5 | 0.6992 | 0.2052 | 0.8727 |
| 10 | 0.8 | 0.6213 | 0.1817 | 0.7612 |
| 10 | 1.0 | 0.6100 | 0.1790 | 0.6557 |
| 10 | 1.5 | 0.6203 | 0.1801 | 0.6312 |
| 100 | 0.5 | 0.3611 | 0.0995 | 1.4312 |
| 100 | 0.8 | 0.3543 | 0.0962 | 1.3742 |
| 100 | 1.0 | 0.3682 | 0.1080 | 1.2377 |
| 100 | 1.5 | 0.4132 | 0.1241 | 1.2125 |

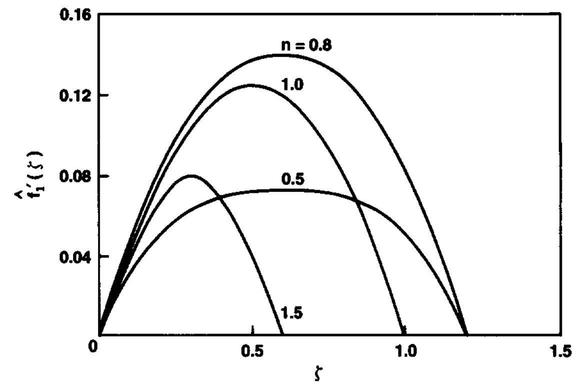


*Figure 10* Velocity profiles $\hat{f}_1(\zeta)$ *for large values of Pr*

**Table 3** $\hat{f}_1''(0)$ *and* $\hat{f}_1^{iv}$ *for large Pr case*

| $n$ | $\hat{f}_1''(0)$ | $\hat{f}_1^{iv}(0)$ |
|---|---|---|
| 0.5 | 1.2341 | 0.7512 |
| 0.8 | 1.1721 | 0.6213 |
| 1.0 | 1.0850 | 0.5403 |
| 1.5 | 1.0741 | 0.5102 |

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